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# Valuation of S&P CNX Nifty Options: Comparison of Black-Scholes and Hybrid ANN Model

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## Abstract

The ability of the Black-Scholes (BS) model and its variants to produce reasonable fair values for options has proven itself for the more than thirty years of its existence. Despite usefulness of BS type models, significant discrepancies remain between the real market prices of options and their theoretical fair values arrived at by using them. This paper investigated the pricing accuracy of a hybrid model, developed by combining the BS model and Artificial Neural Networks (ANNs), for estimating the fair value of options. The relative pricing accuracy of BS model and hybrid model was tested for the options traded at National Stock Exchange of India Ltd., a leading stock exchange in India. The hybrid model was found to outperform BS model on various quantitative parameters viz., mean deviation; mean absolute deviation; mean proportionate deviation; mean squared deviation. Therefore, it is concluded that ANNs can be trained to learn the nonlinear relationship underlying the BS model and hence provide better estimates of fair value of options.

## Introduction

The Black-Scholes (BS) model and its variants postulate that option price is a function of five variables: value of the underlying asset, standard deviation of its expected returns, exercise price of the option, time until the maturity of the option, and interest rate on the default-free bond. The relationship between option price and the five variables is a complex nonlinear one. Although the BS type models rely on several highly questionable assumptions, yet the significant discrepancies remain between the real market prices of options and their estimated fair values arrived at by using them. Due to the above weaknesses of BS type models, attempts have been made to work out the alternative better techniques for option valuation.

The use of Artificial Neural Networks (ANNs) in finance is a recent phenomenon. An ANN is a computational technique inspired by studies of the brain and nervous system. It is an information-processing system designed to mimic the ability of the human brain to comprehend relationships and patterns. They have been found to perform well in a number of applications in which linear models fail to perform well. Specially, when it comes to forecasting financial market variables characterized by non-stationarity, neural networks incorporating nonlinear regression models have a distinct edge. Given that ANNs have been shown to learn complex relationships, several studies have looked at the ability of the neural networks to learn Black-Scholes type model (Hutchinson et al., 1994; Carverhill and Cheuk, 2003; Hamid and Habib, 2005 etc.).

This study extends the above literature and uses a hybrid model for option pricing developed by combining the BS model and ANNs. The relative pricing accuracy of hybrid model and BS model was examined for the options traded at National Stock Exchange (NSE) of India Ltd., a leading stock exchange in India<sup>2</sup>. It was found that the hybrid model provide better estimates of fair value of options than the BS model.

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<sup>2</sup> The NSE introduced options on its index, S&P CNX Nifty, on June 4, 2001. The index is well diversified 50 stock index accounting for 23 sectors of the Indian economy. The NSE accounts for more than 97% of the derivatives trading in India. The total number of trades in 'the index options' have risen fast and have surpassed the trades in 'cash markets', though it is still to reach the volumes of the other international exchanges.

The present study extends the prior literature on option pricing via ANNs in the following ways. First, none of the existing studies have tested the efficacy of ANNs to estimate the pricing of option traded on the Indian stock exchanges<sup>3</sup>. Second, most existing studies, take the input variables for ANNs in their native form. Instead, in this study, transformed variables are fed into the hybrid network. Third, BS model in its native form does not allow for dividends on American options. This paper would allow for dividends in the model. Third, this is the first paper to provide a review of the diffused and fragmented literature related to application of ANNs for option pricing, which is scattered over a wide range of disciplines.

The remaining of this paper is organized as follows. The next section summarizes the existing literature existing on this topic. Section 3 describes BS model and hybrid option pricing model. Section 4 documents the data used, and Section 5 explains how the models were fitted to the data.

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<sup>3</sup> The only documented study found on efficiency of option pricing models in the Indian context was by Rao, Yadav, Bansal and Jain (2004). They compared pricing efficiency of Black-Scholes, GARCH, and Closed-form GARCH Models in the case of S&P CNX Nifty Options. They found that there is error in the option prices predicted by all the three models. But the GARCH models considerably outperforms Black-Scholes model.

## 2. Literature Review

The numerous studies have examined the relative performance of ANNs in pricing equity options in the USA, the UK, Australia, Brazil, France, Germany, Japan and Sweden.

*USA.* Hutchinson *et al.* (1994) compared three ANNs with the Black–Scholes model in pricing American-style call options on S&P500 futures, and found that all three ANNs were superior to Black–Scholes.

Geigle and Aronson (1999) also examined the performance of ANNs in pricing American-style options on S&P500 futures, and found they were superior to Black–Scholes.

Malliaris and Salchenberger (1993) compared the performance of the Black–Scholes model and an ANN in pricing American-style S&P100 call options. They found that Black–Scholes was preferable for in-the-money options, whereas the ANN performed better for out-of-the-money options.

Kitamura and Ebisuda (1998) found that the performance of an ANN in pricing American-style S&P100 call options was poor. However, as well as a very small sample, this result may be due to the use of only two inputs to the ANN. Qi and Maddala (1996) compared the performance of an ANN in pricing European-style call options on the S&P500 index with that of Black–Scholes and concluded that the ANN was superior. A similar conclusion was reached by Garcia and Gençay (1998, 2000), Gençay and Qi (2001), Gençay and Salih (2001), Ghaziri *et al.* (2000), Liu (1996) and Saito and Jun (2000).

Dugas *et al.* (2002) found that constraining the ANN produced better prices for European-style call options on the S&P500 index than those by an unconstrained ANN.

Kelly (1994) priced American-style put options on four US firms using an ANN and the binomial option pricing model. He found that the ANN was clearly more accurate than the binomial model.

**UK.** Niranjana (1996) used daily data from February to December 1994 for call and put FTSE 100 options. He compared the pricing errors for an ANN and Black–Scholes, and for a sample of 100 days found no clear dominance in pricing accuracy. Using the Niranjana (1996) data, De Freitas *et al.* (2000) applied ANNs and Black–Scholes to price FTSE 100 call and put options, and found that all the ANNs considered were superior to the Black–Scholes model.

Healy *et al.* (2002) used closing prices for FTSE 100 call options for 1992–1997 and found that their ANN fitted the data well (there was no direct comparison with the Black–Scholes prices).

**Australia, Brazil, France, Germany, Japan and Sweden.** Lajbcygier *et al.* (1996a,b) compared three ANNs with three closed-form models (Black–Scholes, Barone-Adesi and Whaley and modified Black) in pricing American-style call options on Australian Share Price Index futures. They concluded that the ANNs were inferior to the theory-based models; however, for observations that were near-the-money for short-maturity options, the ANNs were superior.

Lachtermacher and Rodrigues Gaspar (1996) used ANNs to price options on the shares of the Brazilian company Telebrás, and found the ANNs were superior to Black–Scholes. De Winne *et al.* (2001) employed an ANN to price options on French CAC 40 index options, which are American style. They compared their ANN with the binomial model when both models used dividends, and found that their ANN was almost as good as the binomial model.

Anders *et al.* (1998) used data on European-style DAX call options and discovered that the ANN was superior to Black–Scholes, as did Ormoneit (1999) and Krause (1996).

Herrmann and Narr (1997) studied both call and put options on the DAX (both European style), and found that all four ANNs outperformed Black–Scholes. Hanke (1999a) applied ANNs and the Black–Scholes model to European-style call options on the DAX index. After optimizing the volatility and interest rate data to suit the Black–Scholes model, the ANN was less accurate than Black–Scholes.

Yao *et al.* (2000) used ANNs to price call options on Nikkei 225 futures, which are American style, and found they outperformed Black–Scholes. Amilon (2001) compared the performance of an ANN with Black–Scholes in pricing European-style call options on the OMX index. He controlled for dividends by omitting data for the 2 months when shares go ex-dividend in Sweden. For both historical and implied volatilities, the ANN was generally superior.

These papers support the view that ANNs are capable of outperforming well-regarded closed-form models in pricing call options. Futures contracts do not pay dividends, and so this complication was absent from the studies by Geigle and Aronson (1999), Hutchinson *et al.* (1994), Lajbcygier *et al.* (1996) and Yao *et al.* (2000). However, American-style options on a futures contract may be exercised early, as may warrants, and so being American style may be valuable. Since Black–Scholes is only appropriate for pricing European-style options, the outperformance of Black–Scholes by the ANN found by Geigle and Aronson (1999), Ghaziri *et al.* (2000), Hutchinson *et al.* (1994), Lajbcygier *et al.* (1996a,b), Malliaris and Salchenberger (1993a,b) and Yao *et al.* (2000) may be due to the omission of the early exercise option from the theory-based valuation model.

While many studies have considered options on an underlying asset that pays dividends, the theory-based option pricing models used were often not adjusted to incorporate dividends. This will have biased the theory-based models, leading them to overprice call options and underprice put options. Although the ANNs in these studies were usually not supplied with dividend information, they need not have been biased by the omission of dividends to the same extent as the theory-based models.

As well as pricing exchange-traded equity options, ANNs have also been applied to other options.

Hanke (1997) used simulated data to investigate the performance of ANNs in pricing Asian-style call options, and White (1998, 2000) used real and simulated data for European-style call and put options on Eurodollar futures. Raberto *et al.* (2000) used an ANN to price options on German treasury bonds (Bunds), and Karaali *et al.* (1997) used an ANN to price options on an index of the volatility of the \$–DM exchange rate. Taudes *et al.* (1998) considered using ANNs to value real options, and Carelli *et al.* (2000) applied ANNs to pricing \$–DM forex call and put options. ANNs have also been proposed for pricing European-style contingent claims with state-dependent volatility (Barucci *et al.*, 1996, 1997). Provided they are traded on competitive markets for which a price history is available, ANNs have the potential to price a very wide range of financial securities.

### 3. Option Pricing Models

#### 3.1. Black–Scholes

The Black–Scholes call prices were computed using the standard formula, but with the Merton (1973) adjustment for dividends:

$$C = MN(d) - K e^{-rt} N(d - \sigma\sqrt{t}) \quad \text{where} \quad d = \frac{\ln(M/K) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

$M = S e^{-Dt}$  is the Merton adjustment for dividends,  $S$  is the current share price,  $K$  is the exercise (or strike) price,  $r$  is the annual risk-free rate of interest on a continuously compounded basis (e.g. 0.06),  $t$  is the time to expiry in years (e.g. 0.25),  $\sigma$  is the standard deviation of the share's continuously compounded annual rate of return (e.g. 0.30),  $D$  is the annual dividend rate (e.g. 0.05), and  $N(d)$  is the probability that a standardized normally distributed random variable will be less than or equal to  $d$ .

#### 3.2. Artificial Neural Networks<sup>4</sup>

A number of different approaches are classified as members of the ANN family. Our investigation concentrates on the multilayer perceptron (MLP). This is one of the most popular approaches and has been used in the majority of applications to options pricing (e.g. Lajbcygier and Connor, 1997a,b; Anders *et al.*, 1998; White, 1998). It has also been applied successfully to a range of difficult and diverse problems (Brockett *et al.*, 1997; OhnoMachado and Rowland, 1999). Further, Hornik *et al.* (1989, 1990) demonstrated that multilayer feedforward networks are able to approximate a large class of functions and their derivatives accurately with a single hidden layer. A further advantage of feedforward networks is their ability to deal with missing or spurious data.

MLPs consist of connected layers of processing elements, called neurons that pass information through the network by weighted connections. The input variables are presented to the input layer of processing elements, which sends a signal that propagates through the network layer by layer.

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<sup>4</sup> Parts of this section are adapted from Hamid and Habib (2005).

A neural network has (a) processing elements, (b) connections between the elements, (c) weights associated with the connections, (d) activation function.

A neural network can have a large number of processing elements called *neurons* or *nodes* or *cells* or *units* in which information processing takes place. In its simplest form, these neurons are arranged in two layers. The first layer is the input layer -- it takes in inputs to be processed. There will be as many neurons in the input layer as there are input categories. The second layer is the output layer. It will have as many neurons as there are output categories. To process complex problems, the network can have one or more intermediate layers called hidden layers -- so called, because they are essentially hidden from the access of users. The optimal size for the number of layers and neurons per layer is a matter of experimentation. But one hidden layer can essentially map any nonlinear function. The optimal number of neurons in the hidden layer will probably be from one-half to two times the number of neurons in the input layer.

Each neuron has an activation (also called "transfer" or "squashing") function which is applied to the input to determine the output from that neuron. The inputs that a neuron receives from other neurons are summed up and then passed through its transfer function to get the output from that neuron. This function tries to find a relationship between the input variables on one hand and the desired output on the other. The most commonly used transfer functions are the sigmoid, variations of sigmoid, the hyperbolic tangent, and the Gaussian. Thus, a transfer function describes the behavior of a neuron in a neural network. For nonlinear modeling, a transfer function should be nonlinear and continuously differentiable.

In order for the ANN to learn, data on the possible factors influencing the phenomena is required. In the case of option pricing, these factors may be chosen from the inputs required by a corresponding theory-based option pricing model. An ANN does not rely on assumptions concerning the price process of the underlying asset (e.g. constant-volatility geometric Brownian motion), nor does it depend on the specification of theory that connects the price of the underlying asset to the price of the option. Therefore, the

strength of ANNs lies in modeling those relationships between the input and output variables that may be complex and difficult to capture in a convenient mathematical formulation.

Finally, ANNs are flexible and can be used to generate pricing models for a wide variety of options, including options that are difficult to price using the conventional theory-based approach.

The network learns by comparing the resulting output with the desired output and then applying an adjustment to the network weights in accordance with an error correction rule. This is called error back-propagation and is commonly based on the least mean square algorithm.

In order to construct an MLP, various decisions must be made. These are: the number of hidden layers, the number of processing elements in the hidden layer(s), the learning rate and momentum, the set of input variables and the sample period. In addition, preprocessing the inputs before presenting them to the network can reduce the learning required of the network. For example, the ratio of two inputs may be more important in determining an outcome than each input individually. In which case, it is beneficial to generate a new input by dividing these inputs in the preprocessing phase. All the above decisions are key to the success of the MLP.

#### 4. Data

The analysis used data on options traded on NSE over the period from 1 November 2005 to 25 January 2007. Only index options on S&P CNX Nifty index were considered. Stock options are American in nature, whereas Black-Scholes is designed to price only European options correctly. The analysis required ANN to be trained separately on call and put options.

The Black-Scholes model requires values for six parameters: spot price, strike price, maturity, risk-less interest rate, dividend rate and volatility. The daily closing values of the S&P CNX Nifty index were taken from NSE's archives sections, the strike price for each observation was adjusted for dividends and stock splits, and the maturity of each observation (in days) was computed using the date of the observation and the expiry month of the option. As the sample data had an average maturity of 70 days, the annualized risk-less interest rate was measured using the 3 month Treasury bill rate. The daily volatilities were taken as historical volatilities over a period of last one year. This was then annualized before being put into the Black Scholes formula.

## 5. Methodology

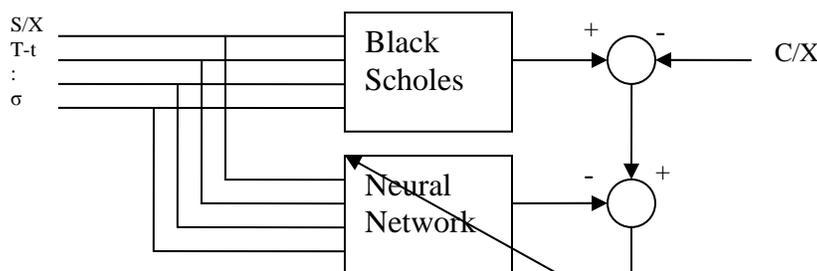
### Hybrid Model

The basis of the hybrid approach to the problem is in using the Black-Scholes model as a base, and allowing the neural network to augment its performance. This can be illustrated as follows:

The project aims to train a hybrid neural network to predict option pricing and then test its validity. The hybrid model combines the Black-Scholes model and ANNs.

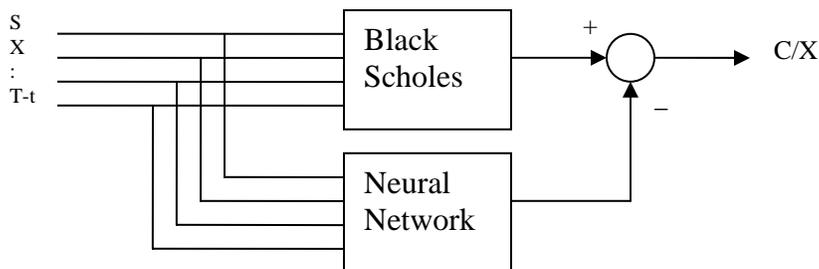
The Black-Scholes model for option pricing has become a de facto standard in the finance industry for a variety of options. The ability of the model to produce reasonable fair values for options has proven itself for the more than thirty years that it has existed. Despite its usefulness, there appear to remain discrepancies between the real market values and the values that a Black-Scholes model would produce. Some scholars have proposed a simple function approximation approach through the use of ANNs, where the network is trained to learn option price data, and it is shown that the neural network approach can achieve comparable performance to (and sometimes better than) the Black-Scholes model.

The basis of the hybrid approach to the problem is in using the Black-Scholes model as a base, and allowing the neural network to augment its performance. This can be illustrated as follows:



Where  $S$  is the value of the underlying asset and  $C$  is the value of the call option on the underlying asset,  $r$  is the risk free rate to the time of the expiration of the option,  $\sigma$  is the standard deviation of the instantaneous rate of return on the underlying asset ( $S$ ),  $X$  is the strike price of the option and  $T-t$  is the time to expiry.

The values of  $S$ ,  $X$ ,  $T-t$  and  $C$  are obtained from past market option information, and the interest rate and volatility can be estimated or approximated as desired. Essentially, the network has  $S/X$ ,  $T-t$ , the interest rate and the volatility presented as inputs, and the difference between the Black-Scholes model and the  $C/X$  value taken from the real data presented as targets. The network is thus trained to produce an appropriate deviation from the Black-Scholes according to the input parameters as shown in the figure above. Therefore, when the system is used for pricing, the difference between the Black-Scholes and the network output should produce the appropriate estimated  $C/X$  value. The network is shown in the figure below.



To estimate the volatility we will use the annualised sample standard deviation of the daily returns of the contract.

$$\text{i.e. } \sigma = s\sqrt{252}$$

where  $s$  is the sample standard deviation of the daily returns of the contract.

The data used will be options traded in S&P CNX Nifty index (alternately, other exchange traded index options depending on the availability of the data can be used).

The above model is developed using SAS enterprise miner. Following is the process model designed to model the option prices:

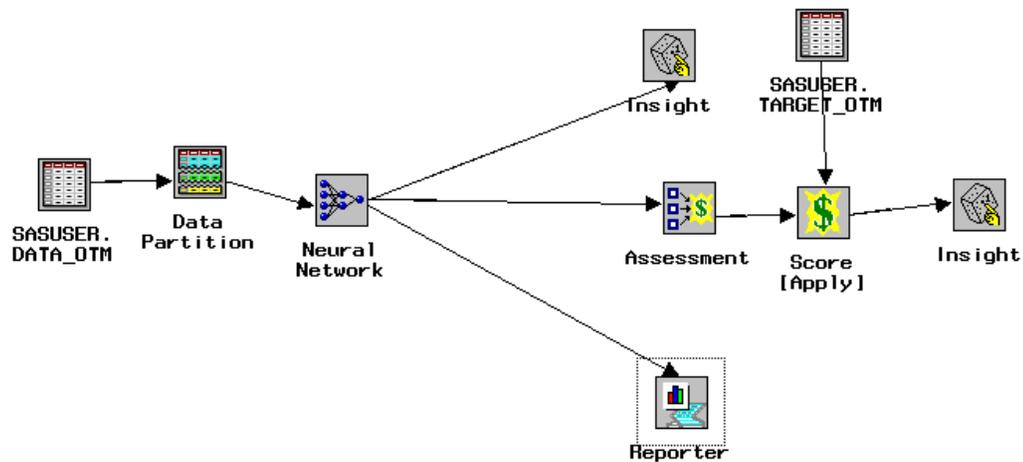


Figure 1: Process flow diagram in SAS Enterprise miner

The results of the hybrid model are compared with the Black-Scholes model and using neural networks alone to test the accuracy of the model and its robustness. The statistical measures for checking the accuracy can be  $R^2$  and NRMSE

### Performance Measurement

As there is no agreement on the appropriate loss function, a single generally accepted measure of the pricing accuracy of Black-Scholes and the MLP in generating the actual call prices is not available. Therefore, five alternative summary measures of performance were used: (a) squared correlation between the actual and computed prices ( $R^2$ ); (b) mean deviation (MD); (c) mean absolute deviation (MAD); (d) mean proportionate deviation (MPD); (e) mean squared deviation (MSD)

The measures used were defined as follow:

$$R^2 = \frac{(n \sum xy - (\sum x)(\sum y))^2}{(n \sum x^2 - (\sum x)^2)(n \sum y^2 - (\sum y)^2)}$$

$$MD = (\sum(y - x))/n$$

$$MAD = (\sum|y - x|)/n$$

$$MPD = (\sum(y - x)/x)/n$$

$$MSD = \sum(y - x)^2/n$$

where  $x$  is the actual value of the dependent variable (e.g.  $C/K$ ),  $y$  is the estimated value of the dependent variable, and  $n$  is the number of observations.

The  $R^2$  measure is a measure of the correlation between the two variables, and hence will represent a closeness of variation of the actual and estimated values. This is useful in verifying the closeness of the relationship between the two variables, but not really the absolute closeness of fit. For this problem, however, it is important that the results be close to the actual data in an absolute sense, something which the NRMSE is more suited to. It measures the root mean square error, and normalizes it to make it independent to scaling effects.

## 6. Results

The entire data set was divided into three parts:

- Training: 40%
- Validation: 30%
- Test: 30%

Training part of the data is used by the SAS package to optimize the neural network. It decides on the number of hidden layers, as well as the number of processing elements (PEs) in each layer.

This is followed by the validation stage, whereby the weights assigned in the neural network are checked on the new data set. Tweaking of parameters is done to optimize on the expected output matching with the actual output.

Finally the model is run on the test data set, to gauge its performance in terms of handling new data.

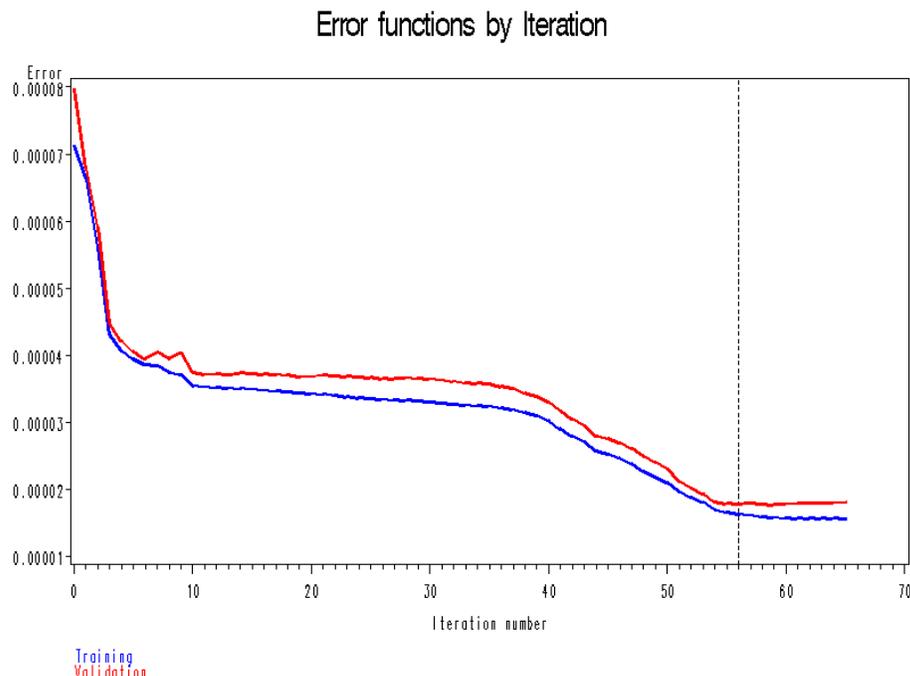


Figure 2: Finding the optimal neural network

On analyzing the data we found that homogeneity hint clearly improves on the MLP's performance relative to when S and K are used as separate inputs. This preprocessing of the data is based on the result from finance theory that S and K can be combined into the variable S/K, which is moneyness. The results presented are typical of all experiments run, with respect to the use of the homogeneity hint.

They also accord with previous studies by other researchers who have used the homogeneity hint and moneyness. The use of moneyness (not S and K) as an input variable, and C/K (not C) as the output variable, is the key to ANNs outperforming Black-Scholes.

Table I. In the money European Style Index options

	<b>MD</b>	<b>MAD</b>	<b>MPD</b>	<b>R<sup>2</sup></b>	<b>MSD</b>
Black Scholes	26.218645	28.60300998	0.177490174	12.1158052	1246.214139
Hybrid model	0.5963087	9.602887973	0.000466767	9.0978350	202.6400226

Table II. Out of money European Style Index options

	<b>MD</b>	<b>MAD</b>	<b>MPD</b>	<b>R<sup>2</sup></b>	<b>MSD</b>
Black Scholes	17.4529	26.52849	0.469645	4.7214054	1242.339
Hybrid model	1.009369	10.36371	0.379611	4.5735771	193.7271

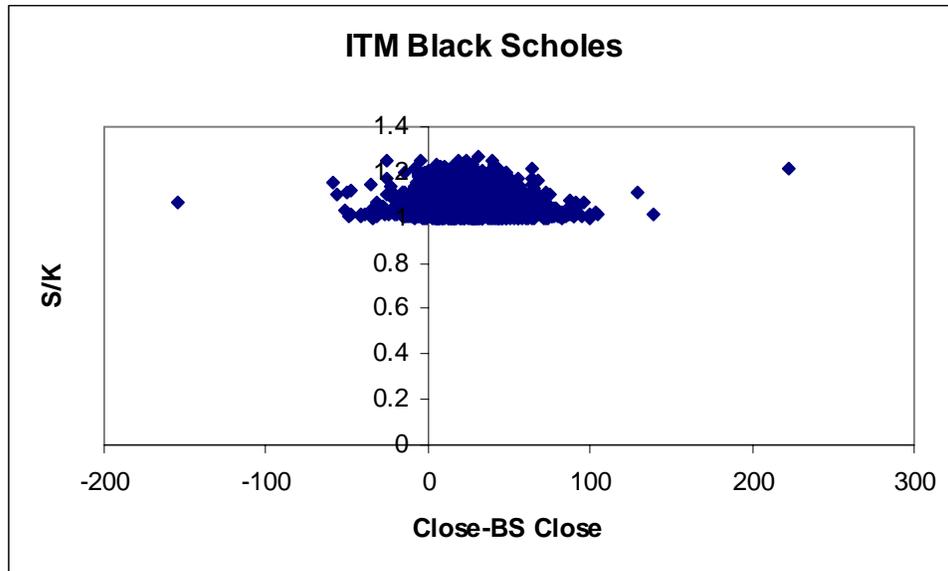


Figure3. Scatter plot of the deviation of the Black- Scholes price from the actual closing price versus moneyness for in the money options

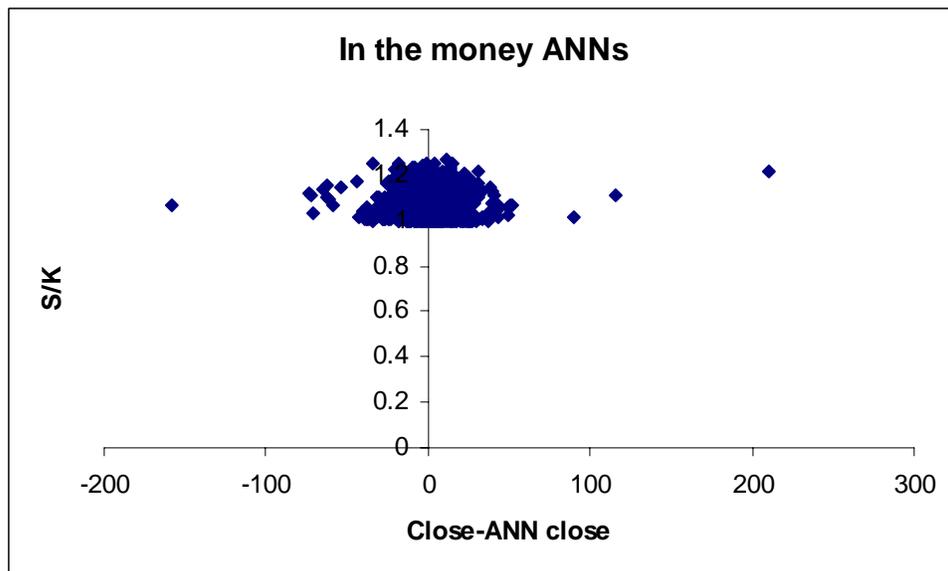


Figure4. Scatter plot of the deviation of the hybrid model price from the actual closing price versus moneyness for in the money options

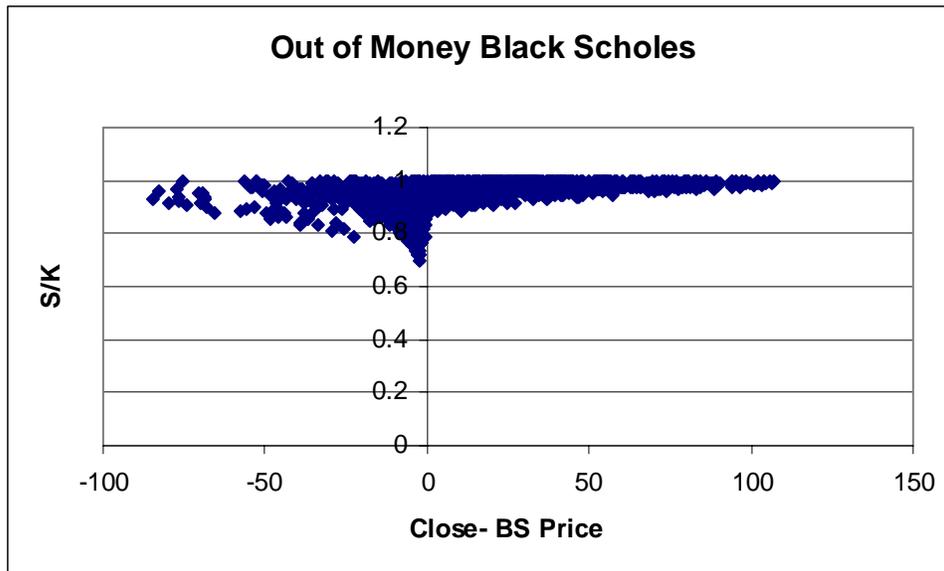


Figure5. Scatter plot of the deviation of the Black- Scholes price from the actual closing price versus moneyness for out of money options

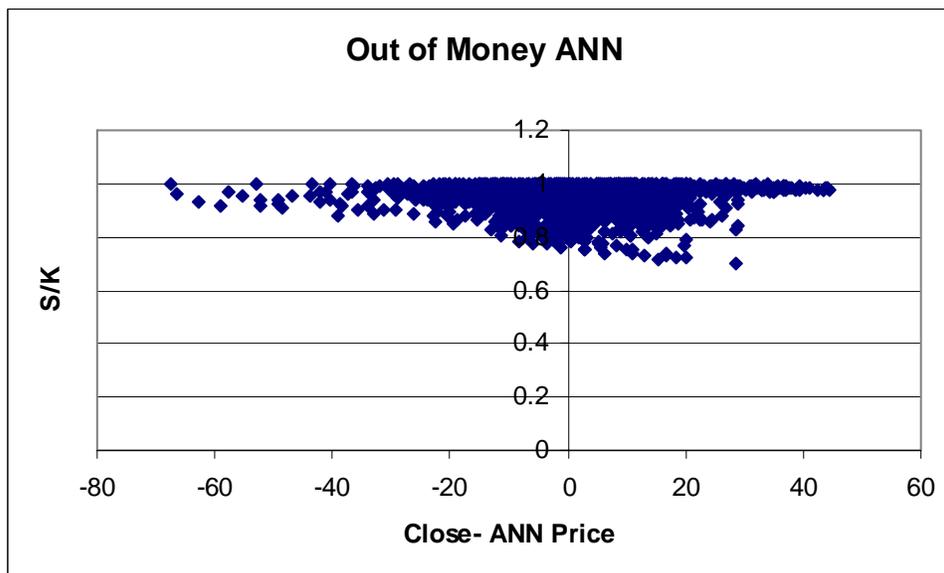


Figure6. Scatter plot of the deviation of the hybrid model price from the actual closing price versus moneyness for out of money options

From the above results, we can infer that our hybrid model outperforms the conventional Black Scholes model in four of the five parameters considered comfortably, both for in-the-money and out-of-the-money options. The only parameter where performance of Black Scholes is better compared to the hybrid model is in the case of  $R^2$ .

## 7. Conclusions

The aim of the paper was to investigate the use of ANNs as a tool for pricing options. In order to evaluate the performance of this approach, European-style equity index options viz., S&P CNX Nifty index options were selected as a case study. The particular advantage in choosing this type of option is that there exists a widely used and highly respected closed-form model (Black and Scholes, 1973) that can be used to benchmark the performance of the ANN. This paper shows that the use of the homogeneity hint and moneyness is of key importance to outperforming Black–Scholes. For both in-the-money and out-of the-money options, the ANN is clearly superior to Black–Scholes. A new hybrid technique has been proposed for option pricing and tested using actual data. Using the Black-Scholes to represent part of the complexity of the actual data, the neural network is freed to learn that part of the real system that cannot be otherwise efficiently represented by conventional models. This hybrid approach is shown to improve the overall performance markedly.

Thus it is concluded that the ANN approach is generally superior to Black–Scholes in pricing S&P CNX Nifty index call options. This is a surprising result, given that European-style equity options are the home ground of Black–Scholes, and suggests that ANNs may have an important role to play in pricing other options for which there is either no closed-form model or the closed-form model is less successful than Black–Scholes for equity options.

The approach presented here has successfully learnt the differences between the actual market determined call prices, and theoretically estimated Black-Scholes call price values. The out-of-sample performance indicates that these differences are not noise, but can be attributed to some systematic deviation. It is possible that these deviations come from a breakdown in the assumptions made in the derivation of the Black-Scholes model. For example, Black and Scholes assumed that the interest rate and volatility were constant over the life of the option. This is evidently not the case, and the Black Scholes model itself is given these varying values as inputs while calculating the estimated

values. The neural network, taking these two parameters as inputs makes no such assumptions and is thus better equipped to incorporate their effects. Geometric Brownian motion in the underlying is another assumption which has been shown to be untrue

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